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DETERMINATION OF THE LOCAL ANGULAR COEFFICIENT FOR A CYLINDRICAL SURFACE

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Algebraic expressions are obtained and graphs are presented for the determination of the local angular coefficient for a particular orientation of an elementary area relative to a cylinder.

In engineering practice when calculating radiant heat transfer one usually uses the approximation where the problem of determining the angular coefficient between a cylindrical surface (F_2) and a structural surface (F_1) comes down to the problem of determining the angular coefficient between the surface F_2 and a surface of elementary area dF_1.

Let us consider the case when the normal to the center of the elementary area is parallel to the axis of the cylinder.*

The algebraic expression for calculating the local angular coefficient is sought by the method of integration of the basic equation [2]

$$\varphi_{dF_1 \cdot F_2} = \int\limits_F \frac{\cos\theta_1 \cdot \cos\theta_2}{\pi r^2} dF_2, \quad (1)$$

which is written in the following form, using $dF_2 = Rd\Phi dy$, $\cos\theta_1 = y/r$, $\cos\theta_2 = (x_1\cos\Phi - R)/r$ and $r^2 = R^2 + x_1^2 + y^2 - 2x_2R\cos\Phi$ for the geometry shown in Fig. 1a:

$$\varphi_{dF_1 \cdot F_2} = \frac{2R}{\pi} \int\limits_0^{\Phi_0} (x_1 \cos\Phi - R)\, d\Phi \int\limits_0^{y_1} \frac{ydy}{(y^2+k)^2}, \quad (2)$$

where $k = R^2 + x_1^2 - 2x_1R\cos\Phi$ and $\Phi_0 = \arccos(R/x_1)$.

The integration of Eq. (2) encounters no mathematical difficulties and is carried out using tables of integrals:

$$\varphi_{dF_1 \cdot F_2} = \frac{1}{\pi} \operatorname{arctg}\left(\frac{x_1+R}{x_1-R} \operatorname{tg}\frac{\Phi_0}{2}\right) + \frac{1}{\pi} \frac{R^2 - x_1^2 - y_1^2}{\sqrt{AB}} \operatorname{arctg}\left(\sqrt{\frac{A}{B}} \operatorname{tg}\frac{\Phi_0}{2}\right), \quad (3)$$

where $A = y_1^2 + (x_1 + R)^2$ and $B = y_1^2 + (x_1 - R)^2$.

The limiting local angular coefficient is of interest in some cases when the height of the cylinder approaches infinity. From Eq. (3) it follows that

*The solution of the problem for the case when the normal to the center of the elementary area is perpendicular to the axis of the cylinder is presented in [1].

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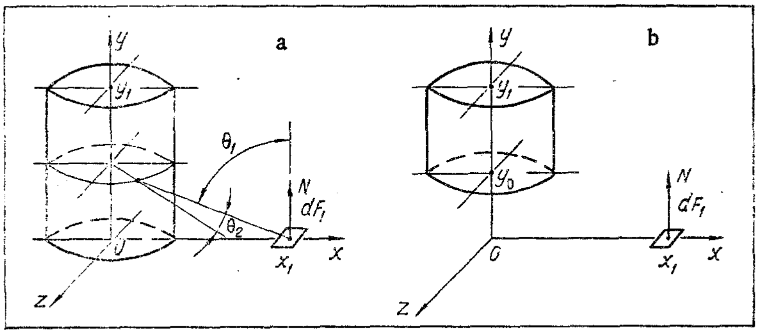

Fig. 1. Basic (a) and supplementary (b) systems for calculating the angular coefficient of an elementary area to a cylindrical surface.

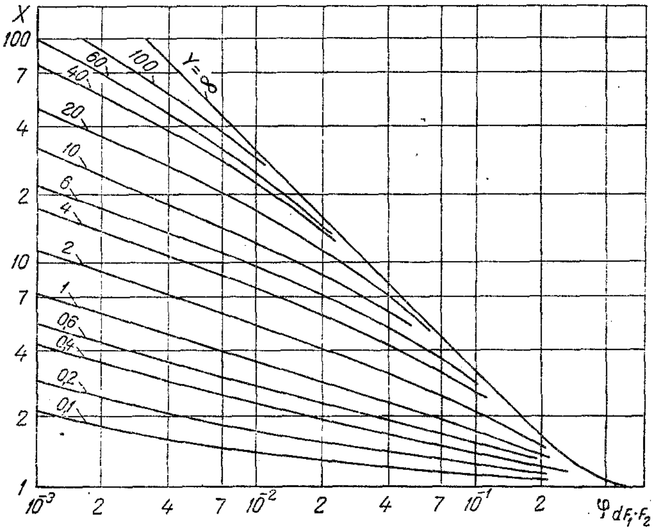

Fig. 2. Graph of dependence of local angular coefficient on height of cylinder and its distance from center of elementary area.

$$\lim_{y_1 \to \infty} \varphi_{dF_1 \cdot F_2} = \frac{1}{\pi} \operatorname{arctg} \left(\frac{x_1 + R}{x_1 - R} \operatorname{tg} \frac{\Phi_0}{2} \right) + \frac{1}{\pi} \operatorname{arctg} \left(\operatorname{tg} \frac{\Phi_0}{2} \right). \tag{4}$$

The solutions in the form of (3) and (4) can serve as the initial equation for obtaining solutions when there is partial shading of the surface F_2 of the cylinder along its generatrix seen from the center of the area dF_1.

For the geometry under consideration, using

$$\operatorname{tg} \frac{\Phi_0}{2} = \sqrt{\frac{x_1 - R}{x_1 + R}}$$

the solutions (3) and (4) are written in the following final form, respectively:

$$\varphi_{dF_1 \cdot F_2} = \frac{1}{\pi} \operatorname{arctg} \sqrt{\frac{x_1 + R}{x_1 - R}} + \frac{1}{\pi} \frac{R^2 - x_1^2 - y_1^2}{\sqrt{AB}} \operatorname{arctg} \sqrt{\frac{A(x_1 - R)}{B(x_1 + R)}}, \tag{5}$$

$$\lim_{y_1 \to \infty} \varphi_{dF_1 \cdot F_2} = \frac{1}{\pi} \operatorname{arctg} \sqrt{\frac{x_1 + R}{x_1 - R}} + \frac{1}{\pi} \operatorname{arctg} \sqrt{\frac{x_1 - R}{x_1 + R}}. \tag{6}$$

The sole restriction imposed on the solution of the problem is the condition according to which $x_1 \geq R$ and $y_1 > 0$.

At the boundary of the cylinder ($x_1 = R$) the solutions (5) and (6) are reduced to the form $\varphi_{dF_1 \cdot F_2} = 0.5$.

In the case of the system when the base of the cylinder is raised above the plane of the elementary area by an amount y_0 (Fig. 1b) the local angular coefficient is determined from the equation

$$\varphi_{dF_1 \cdot F_2} = \varphi^{*}_{dF_1 \cdot F_2} - \varphi^{**}_{dF_1 \cdot F_2}, \qquad (7)$$

where $\varphi^{*}_{dF_1 \cdot F_2}$ and $\varphi^{**}_{dF_1 \cdot F_2}$ are the local angular coefficients for cylinders of heights y_1 and y_0.

Curves of the dependence of the local angular coefficient on the dimensionless distance $X = x_1/R$ and the dimensionless height $Y = y_1/R$, obtained on the basis of the solutions (5) and (6), are presented in Fig. 2.

NOTATION

dF_1, elementary area; F_2, section of emitting surface seen from center of area dF_1; $\varphi dF_1 dF_2$, local angular coefficient; R, radius of cylinder; x_1, distance from center of area dF_1 to axis of cylinder ($X = x_1/R$, dimensionless distance); y_1 and y_0, height of cylinder ($Y = y/R$, dimensionless height); r, distance of center of area dF_1 from an arbitrary point on surface F_2; N, normal to center of area dF_1; Φ_0, angle defining boundary of visible section of cylindrical surface; θ_1, angle between normal to center of area dF_1 and straight line connecting center of dF_1 with an arbitrary point on surface F_2; θ_2, angle between normal to surface F_2 at an arbitrary point and straight line connecting this point with center of dF_1; x, y, z, coordinates.

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METHOD OF "JOINING" OF SOLUTIONS IN THE DETERMINATION OF A PLANE AND A CYLINDRICAL PHASE INTERFACE IN THE STEFAN PROBLEM

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It is shown that the position of the phase interface in the Stefan problem can be expressed through two functions: One function determines the position of the melting-temperature isotherm in the problem without phase transitions and the second does not depend on time.

Two most popular courses presently exist for determining the law of motion of the phase interface in the Stefan problem: approximate analytical solutions of the problem and numerical methods using a computer. Considerable difficulties are encountered on the latter course in the stage of analysis of the numerical material obtained and the attempt to represent the results in the form of analytical dependences of the position of the phase interface on the determining parameters and criteria. The following method can be proposed as one variant of analysis. Let the amount of latent heat of the phase transitions per unit volume of material approach zero. In this case a plane interface, for example, approaches its limiting value

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